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PAR-003-001544

Seat No. ______

B. Sc. (Sem. V) Examination

October / November - 2018

Statistics : S-503

(Statistical Inference)

(Old Course)

Faculty Code : 003

Subject Code : 001544

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :
(1) All the questions are compulsory.
(2) Que-1 carry 20 marks.
(3) Que-2 and 3 carry 25 marks.
(4) Students can use their own Scientific calculators.

1 Filling the blanks and short questions : (Each 1 mark) **20**

(1) The estimation of a parameter by the method of minimum Chi-square utilizes ________ statistic.

(2) An estimator T_n which is most concentrated about parameter θ is the _______ estimator.

(3) If T_n is an estimator of a parametric function $\tau(\theta)$, the mean square error of T_n is equal to _______.

(4) If a random sample $x_1, x_2, x_3, ..., x_n$ is drawn from a population $N(\mu, \sigma^2)$, the maximum likelihood estimate of μ is _______.

(5) If T_n is an estimator of a parameter θ of the density $f(x;\theta)$ the quantity $E\left[\frac{\partial}{\partial\theta}\log f(x;\theta)\right]^2$ is called the ________.

(6) If $S = s(X_1, X_2, X_3, ..., X_n)$ is a sufficient statistic for θ of density $f(x;\theta)$ and $f(x_i;\theta)$ for i =1, 2, 3...., n can be factorised as $g(s,\theta)h(x)$, then $s(X_1, X_2, X_3, ..., X_n)$ is a ________.

(7) An estimator of $v_\theta(T_n)$ which attains lower bound for all θ is known as ________.

(8) If an estimator T_n converges in probability to the parametric function $\tau(\theta), T_n$ is said to be a ________ estimator.

(9) An unbiased and complete statistic is compulsorily ________.

(10) Let $x_1, x_2, x_3, ..., x_n$ be a random sample from a density $f(x,\theta) = \theta e^{-\theta x}$. Then the Crammer-Rao lover bound of variance of unbiased estimator is ________.

(11) __________ is an unbiased estimator of p^2 in Binomial distribution.

(12) Method of moments for estimating the parameters of a distribution was given by ________ in 1894.

(13) A value of an estimator is called an ________.

(14) If we have a random sample of size n from a population $N(\mu, \sigma^2)$, then sample mean is __________ efficient than sample median.

(15) Let there be a sample of size n from a normal population with mean μ and variance σ^2. The efficiency of median relative to the mean is ________.

(16) Write likelihood function of Poisson distribution.

(17) Name different criteria of good estimators.

(18) Write likelihood function of

$$f(x, \theta) = \binom{-k}{x} \theta^k (\theta - 1)^x ; 0 \leq \theta \leq 1.$$

(19) Define Likelihood function.

(20) Obtain Crammer-Rao lower bound of variance of unbiased estimator of parameter of $f(x, \theta) = \theta e^{-x\theta}; 0 \leq x \leq \infty$.

2 (a) Write the answer any three each 2 marks) **6**

(1) Define Sufficiency.

(2) Define Efficiency.

(3) Define Parameter space.

(4) Define Uniformly Most Powerful Test (UMP test).

(5) Define ASN function of SPRT.

(6) Obtain likelihood function of Negative Binomial distribution.

(b) Write the answer any three each 3 marks) **9**

(1) Obtain estimator of θ by method of moments in the following distribution

$f(x; \theta) = \theta e^{-\theta x}$; where $0 \leq x \leq \infty$.

(2) $\frac{\overline{x}}{n}$ is a consistent estimator of p for Binomial distribution.

(3) Obtain MVUE of parameter θ for Poisson distribution. Also obtain its variance.

(4) Obtain Operating Characteristic (OC) function of SPRT.

(5) Give a random sample $x_1, x_2, x_3, \ldots\ldots, x_n$ from distribution with p.d.f $f(x;\theta) = \frac{1}{\theta}; 0 \le x \le \theta$. Obtain power of the test for testing $H_0 : \theta = 1.5$ against $H_1 : \theta = 2.5$, where $C = \{x; x \ge 0.8\}$.

(6) Obtain unbiased estimator of $\frac{kq}{p}$ of Negative Binomial distribution.

(c) Write the answer any two each 5 marks) **10**

(1) Obtain OC function for SPRT of Binomial distribution for testing $H_0 : p = p_0$ against $H_1 : p = p_1 (> p_0)$.

(2) Estimate α and β in the case of Gamma distribution by the method of moments $f(x;\alpha,\beta) = \frac{\alpha^{\beta}}{\Gamma\beta} e^{-ax} x^{\beta - 1}; x \ge 0, \alpha \ge 0$.

(3) State Crammer-Rao inequality and prove it.

(4) If T_1 and T_2 be two unbiased estimator of θ with variance σ_1^2, σ_2^2 and correlation ρ, what is the best unbiased linear combination of T_1 and T_2 and what is the variance of such a combination?

(5) Obtain Likelihood Ration Test :

Let $x_1, x_2, x_3, \ldots\ldots, x_n$ random sample taken from $N(\mu, \sigma^2)$. To test $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 \ne \sigma_0^2$.

3 (a) Write the answer any three each 2 marks : **6**

(1) Show that Σx_i is a sufficient estimator of θ for Geometric distribution.

(2) Define Minimum Variance Bound Estimator (MVBE).

(3) Define Consistency.

(4) Obtain an sufficient estimator of θ by for the following distribution $f(x;\theta) = \theta^x (1-\theta)^{(1-x)}; x = 0, 1$.

(5) Obtain an unbiased estimator of θ by for the following distribution $f(x;\theta) = \frac{1}{\theta}; 0 \le x < \theta$.

(6) Define Complete family of distribution.

(b) Write the answer any three each 3 marks) **9**

(1) Obtain MLE of parameter p for the following distribution $f(x;p) = pq^x; x = 0, 1, 2, ..., \infty$.

(2) Let $x_1, x_2, x_3, ..., x_n$ be random sample taken from $N(\mu, \sigma^2)$, then find sufficient estimator of μ and σ^2.

(3) Prove that $E\left(\frac{\partial \log L}{\partial \theta}\right)^2 = -E\left(\frac{\partial^2 \log L}{\partial \theta^2}\right)$

(4) Let p be the probability that coin will fall head in a single toss in order to test $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{3}{4}$. The coin is tossed 6 times and H_0 is rejected if more than 4 head are obtained. Find the probability of type-I error, type-II error and power of test.

(5) Use the Neyman Pearson lemma to obtain the best critical region for testing $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda = \lambda_1$ in the case of Poisson distribution with parameter λ.

(6) Obtain an unbiased estimator of population mean of χ^2 distribution.

(c) Write the answer any two each 5 marks : **10**

(1) Construct SPRT of Poisson distribution for testing $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda = \lambda_1 (> \lambda_0)$. Also obtain OC function of SPRT.

(2) State Neyman-Pearson lemma and prove it.

(3) Let $x_1, x_2, x_3, ..., x_n$ be random sample from the p.d.f. $f(x, p) = \frac{1}{(1-q^3)} \binom{3}{x} p^x q^{3-x}$, where $x = 0, 1, 2, 3$.

Estimate parameter of p by the method of moment.

(4) For the double Poisson distribution

$$p(X=x)=\frac{1}{2}\frac{e^{m_1}m_1^x}{x!}+\frac{1}{2}\frac{e^{-m_2}m_2^x}{x!}\ ;0,1,2,.....$$

Show that the estimator for m_1 and m_2 by the method of moment are $\mu_1^{'}\pm\sqrt{\mu_2^{'}-\mu_1^{'}-\left(\mu_1^{'}\right)^2}$.

(5) Obtain MVBE of σ^2 for Normal distribution.